Cálculo Diferencial e Integral: um KIT de sobrevivência

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<u>Integrais de Superfície</u>

Vamos ilustrar o uso do Maple no cálculo de integrais de superfície. Consideremos o caso do cálculo da integral uma função vetorial sobre uma superfície.

Exemplo 1

Exemplo 1 : Seja $F(x, y, z) = (x, y, z)$ um campo vetorial e a superfície parametrizada por $x = \cos(u)\sin(v)$

$y = \sin(v)\sin(u)$ e $z = \cos(v)$, onde u pertence a [$0, 2\pi$] e v pertence a [$0, \pi$].

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> with(linalg):
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```
> F := vector([x,y,z]);
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$$F := [x, y, z]$$

```
> R := vector([cos(u)*sin(v),sin(u)*sin(v),cos(v)]);
```

$$R := [\cos(u)\sin(v), \sin(u)\sin(v), \cos(v)]$$

```
> Ru := map(diff,R,u);
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$$Ru := [-\sin(u)\sin(v), \cos(u)\sin(v), 0]$$

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> Rv := map(diff,R,v);
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$$Rv := [\cos(u)\cos(v), \sin(u)\cos(v), -\sin(v)]$$

O vetor normal a superfície é dada por

> **NormalS := crossprod(Ru,Rv);**

$$NormalS := [-\cos(u)\sin(v)^2, -\sin(u)\sin(v)^2, -\sin(u)^2\sin(v)\cos(v) - \cos(u)^2\sin(v)\cos(v)]$$

> **NormalS := map(simplify,NormalS);**

$$NormalS := [-\cos(u) + \cos(u)\cos(v)^2, -\sin(u) + \sin(u)\cos(v)^2, -\sin(v)\cos(v)]$$

> **H := subs(x=R[1],y=R[2],z=R[3],dotprod(NormalS,F));**

$$H := (-\cos(u) + \cos(u)\cos(v)^2)\cos(u)\sin(v) + (-\sin(u) + \sin(u)\cos(v)^2)\sin(u)\sin(v) - \sin(v)\cos(v$$

> **H := simplify(H);**

$$H := -\sin(v)$$

> **Int(Int(H,u=0..2*Pi),v=0..Pi)=int(int(H,u=0..2*Pi),v=0..Pi);**

$$\int_0^{\pi}\int_0^{2\pi} -\sin(v)\, du\, dv = -4\pi$$

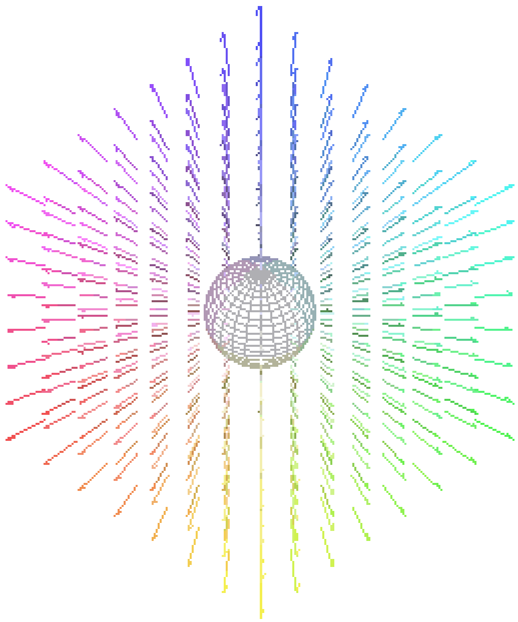
Visualizamos o campo e a superfície

> **with(plots):**

> **campo:=fieldplot3d(F,x=-3..3,y=-3..3,z=-3..3):**

> **superf:=plot3d([cos(u)*sin(v),sin(u)*sin(v),cos(v)],u=0..2*Pi,v=0..Pi):**

> **display({campo,superf});**



Veja o outro Exemplo.

Exemplo 2

O campo é $F := [x, y, -2z]$ e a parametrização da superfície é $R := [a\cos(u)\cos(v), a\sin(u)\cos(v), a\sin(v)]$

onde u pertence a [$0, \frac{\pi}{2}$] e v pertence a [$0, 2\pi$].

> **F := vector([x,y,-2*z]);**

$$F := [x, y, -2z]$$

> **R := vector([a*cos(u)*cos(v),a*sin(u)*cos(v),a*sin(v)]);**

$$R := [a\cos(u)\cos(v), a\sin(u)\cos(v), a\sin(v)]$$

> **Ru := map(diff,R,u);**

$$Ru := [-a\sin(u)\cos(v), a\cos(u)\cos(v), 0]$$

> **Rv := map(diff,R,v);**

$$Rv := [-a\cos(u)\sin(v), -a\sin(u)\sin(v), a\cos(v)]$$

> **NormalS := crossprod(Ru,Rv);**

$$NormalS := [a^2\cos(u)\cos(v)^2, a^2\sin(u)\cos(v)^2, a^2\sin(u)^2\cos(v)\sin(v) + a^2\cos(u)^2\cos(v)\sin(v)$$

> **NormalS := map(simplify,NormalS);**

$$NormalS := [a^2\cos(u)\cos(v)^2, a^2\sin(u)\cos(v)^2, a^2\cos(v)\sin(v)]$$

> **H := subs(x=R[1],y=R[2],z=R[3],dotprod(NormalS,F));**

$$H := a^2\cos(u)\cos(v)^2\,\overline{a\cos(u)\cos(v)} + a^2\sin(u)\cos(v)^2\,\overline{a\sin(u)\cos(v)} - 2a^2\cos(v)\sin(v)\,\overline{a\sin(v}$$

> **H := simplify(H);**

$$H := a\cos(v)\,|a\cos(u)\cos(v)|^2 + a\cos(v)\,|a\sin(u)\cos(v)|^2 - 2a\cos(v)\,|a\sin(v)|^2$$

> **Int(Int(H,u=0..Pi/2),v=0..2*Pi)=int(int(H,u=0..Pi/2),v=0..2*Pi);**

$$\int_0^{2\pi}\int_0^{\frac{1}{2}\pi} -\sin(v)\,du\,dv = 0$$

> **with(plots):**

Vamos visualizar o campo e a superfície juntos

> **campo:=fieldplot3d(F,x=-3..3,y=-3..3,z=-3..3):**

> **superf:=plot3d([2*cos(u)*cos(v),2*sin(u)*cos(v),2*sin(v)],u=0..Pi/2,v=0..2*Pi):**

> **display({campo,superf});**